# UE Modeling and control of PDE

Level
Baccalaureate +5

ECTS
6 credits

Component
UFR PhITEM (physique, ingénierie, terre, environnement, mécanique)

Semester
Automne

- **Teaching language(s):** English
- **Open to exchange students:** Yes
- **Code d'export Apogée:** PAX9SCAH

# Presentation

## Description

This set of courses proposes an overview of recent techniques for the identification, observation, simulation and control of distributed parameter systems. This class of systems is widely used in physics and considered in many applications (such as in environment dynamics, airflow control, structural mechanics, and adaptive optics) having a large or an infinite number of degrees of freedom. A Partial Differential Equation (PDE) usually models them. Their mathematical study asks for a special care to analyze the dynamics behavior and to describe their control properties. Different aspects of this description are considered in this Teaching Unit, by emphasizing the practical methods allowing for some real applications.

This Teaching Unit is composed by three different courses:

## Analysis and control (13.5 h)

| Lesson | Topic |
|---|---|
| 1 | **Some recalls in the analysis of PDE** |
| | *Differential calculus; derivation of a PDE; classification of infinite dimensional systems.* |
| 2 | **Semigroup theory** |

| | |
|---|---|
| | *Strongly continuous semigroups; contraction semigroups.* |
| 3 | **Control and Observation of some particular PDEs** |
| | *Transport equation; heat equation.* |
| 4 | **Stability and Stabilization** |
| | *Definitions; Lyapunov functions.* |

## Modeling and Inverse problems (13.5h)

| Lesson | Topic |
|---|---|
| 1 | **Discretization methods for the numerical approximation of PDEs** |
| | *basics of finite difference and finite element methods; stability analysis for evolution equations.* |
| 2 | **Identification and inverse problems** |
| | *basics of optimization algorithms; derivation of the adjoint of a discretized model; some practical aspects of the derivation of a numerical model.* |
| 3 | **Link with the linear statistical estimation** |

## Distributed optimization (13.5h)

| Lesson | Topic |
|---|---|
| 1 | **Open-loop optimal control of PDE** |
| | *Adjoint-based method for some particular PDEs: a parabolic and a hyperbolic PDE case studies; a short introduction to numerical methods for the solution of open-loop infinite-dimensional optimal control problems.* |
| 2 | **Optimal control of PDE with state-feedback** |
| | *The Linear Quadratic Regulator; solution via the operator Riccati equation; two case studies.* |
| 3 | **Robust control of PDE with state-feedback** |
| | *A game-theoretic approach: the Hinfinity optimal regulator; solution via the associated operator Riccati equation; one case study.* |

*Prerequisites:* basic mathematical background, control theory of finite dimensional systems (control and observation theory for linear ODEs, in particular optimal LQ regulation)

- Schedule
- Modeling
- Control
- Optimization
- Communication networks
- Projects & seminars
- Public speaking

## Course parts

| | | |
|---|---|---|
| UE Modeling and control of PDE - CM | Lectures (CM) | 42h |

**Period :** Semester 9

## Bibliography

### Analysis and control


- J.-M. Coron, "Control and nonlinearity", *Mathematical Surveys and Monographs*, 136, 2007.
- R. F. Curtain and H. Zwart, "An Introduction to Infinite-Dimensional Linear Systems Theory", vol. 21 of Texts in Applied Mathematics, *Springer-Verlag*, New York, 1995.
- Z.-H. Luo, B.-Z. Guo, and O. Morgul, "Stability and Stabilization of Infinite Dimensional Systems with Applications", *Communications and Control Engineering*, Springer-Verlag, London, 1999.
- A. Pazy, "Semigroups of Linear Operators and Applications to Partial Differential Equations", *Springer-Verlag*, New York, 1983.
- M. Tucsnak and G. Weiss, "Observation and Control for Operator Semigroups", *Birkhäuser*, 2009.


### Modeling and Inverse problems


- Allaire G., "Analyse numérique et optimisation", *Les éditions de l'Ecole Polytechnique*, 2005.
- Euvrard D., "Résolution numérique des équations aux dérivées partielles". *Masson*, 1987.
- Larrouturou B. et P.-L. Lions, 1992: "Méthodes mathématiques pour les sciences de l'ingénieur: optimisation et analyse numérique", *Cours de l'Ecole Polytechnique*, 1992.
- Lascaux P. et R. Théodor, "Analyse numérique matricielle appliquée à l'art de l'ingénieur", *Masson*, 1986.
- Rodgers C., "Inverse methods for atmospheric sounding", *World Scientific*, Series on Atmospheric Oceanic and Planetary Physics, 2000.
- Tarantola A., "Inverse problem theory and methods for model parameter estimation", *SIAM*, 2005.


### Distributed optimization


- R. F. Curtain and H. Zwart, "An Introduction to Infinite-Dimensional Linear Systems Theory", vol. 21 of Texts in Applied Mathematics, *Springer-Verlag*, New York, 1995.
- E. Casas, "Optimal Control of PDEs", website .
- A. Bensoussan and P. Bernhard, "On the standard problem of Hinfinity-optimal control problems for infinite-dimensional systems", Identification and control in systems governed by PDEs, pp. 117-140, *SIAM*, Philadephia (PA), 1993.

• R. Curtain, A.M. Peters and B. Van Keulen, "Hinfinity-control with state-feedback: the infinite-dimensional case", *Journal of Math. Syst. Estim. Control*, vol. 3, nb. 1, pp. 1-39, 1993.

# Useful info

## Place

Grenoble

## Campus

Grenoble - Scientific Polygon